Magnetic Moment of The Θ^+ Pentaquark State

Peng-Zhi Huang, Wei-Zhen Deng, Xiao-Lin Chen and Shi-Lin Zhu

Department of Physics, Peking University, Beijing 100871, China

February 1, 2008

Abstract

We have calculated the magnetic moment of the recently observed Θ^+ pentaquark in the framework of the light cone QCD sum rules using the photon distribution amplitudes. We find that $\mu_{\Theta^+} = (0.12 \pm 0.06)\mu_N$, which is quite small. We also compare our result with predictions of other groups.

PACS number: 12.39.Mk, 12.38.Lg, 12.40.Yx

1   Introduction

Early this year LEPS Collaboration at the SPring-8 facility in Japan observed a sharp resonance Θ^+ at 1.54 ± 0.01 GeV with a width smaller than 25 MeV and a statistical significance of 4.6σ in the reaction $\gamma n \to K^+ K^- n$ [1]. This resonance decays into $K^+ n$, hence carries strangeness $S = +1$ and $B = +1$.

Later the same resonance was confirmed by several other groups. In a different reaction $K^+ Xe \to \Theta^+ Xe' \to K^0 p Xe'$, DIANA Collaboration at ITEP observed this resonance at 1539 ± 2 MeV with a width less than 9 MeV [2]. Now Θ^+ decays into $K^0 p$. The confidence level is 4.4σ.

CLAS Collaboration in Hall B at JLAB observed Θ^+ in the $K^+ n$ invariant mass spectrum at 1542 ± 5 MeV in the exclusive measurement of the $\gamma d \to K^+ K^- p n$ reaction [3]. The statistical significance is 5.3σ. The measured width is 21 MeV, consistent with CLAS detector resolution. There was only preliminary hint that Θ^+ might be an iso-singlet from the featureless $M(K^+ p)$ spectrum in the CLAS measurement.

SAPHIR Collaboration observed this positive-strangeness resonance in the nK^+ invariant mass distribution with a 4.8σ confidence level in the photoproduction of the $nK^+K^0_s$ final state with the SAPHIR detector at the Bonn ELectron Stretcher Accelerator ELSA [4]. Its mass is found to be $M_{\Theta^+} = 1540 \pm 4 \pm 2$ MeV. An upper limit of $\Gamma_{\Theta^+} < 25$ MeV was set for the width of this resonance at 90% confidence level. From the absence of a signal in the pK^+ invariant mass distribution in $\gamma p \to p K^+ K^-$ at the expected strength they further concluded that the Θ^+ must be an isoscalar.

Recently another very important observation came from the high energy collision experiment. NA49 Collaboration found evidence for the existence of a narrow $\Xi^- \pi^-$ baryon

1

resonance with mass of (1.862 ± 0.002) GeV and width below the detector resolution of about 0.018 GeV in proton-proton collisions at $\sqrt{s} = 17.2$ GeV [5]. The quantum number of this state is $Q = -2, S = -2, I = 3/2$ and its quark content is $(dsds\bar{u})$. They also observed signals for the $Q = 0$ member of the same isospin quartet with a quark content of $(dsus\bar{d})$ in the $\Xi^- \pi^+$ spectrum. The corresponding anti-baryon spectra also show enhancements at the same invariant mass.

Due to the complicated infrared behavior of Quantum Chromodynamics (QCD), it's nearly impossible to predict the hadron spectrum analytically from first principle. Lattice simulation may provide an alternate feasible way to extract the whole spectrum in the future. But now, people have just been able to understand the first orbital and radial excitation of the nucleon on the lattice [6].

Experimentally there have accumulated tremendous data in the low energy sector in the past decades. Under such a circumstance, various QCD-inspired models were proposed. Among them, the simple quark model (QM) has been surprisingly successful in the classification of hadrons and calculation of their spectrum and other low-energy properties [7]. According to QM, mesons are composed of a pair of quark and anti-quark while baryons are composed of three quarks. Both mesons and baryons are color singlets. Nearly all the experimentally observed hadrons fit into the quark model classification scheme quite nicely.

In contrast, QCD itself does allow the existence of the non-conventional hadrons with the quark content other than $q\bar{q}$ or qqq, which is beyond conventional mesons and baryons in the quark model. Some examples are glueballs $(gg, ggg, \cdots)$, hybrid mesons $(q\bar{q}g)$, and other multi-quark states $(qq\bar{q}\bar{q}, qqqq\bar{q}, qqq\bar{q}\bar{q}\bar{q}, qqqqqq, \cdots)$. In fact, hybrid mesons are found to mix freely with conventional mesons in the large N_c limit [8]. However, despite extensive experimental searches in the past two decades, none of these states has been firmly established until this year [9].

The surprising discovery of very narrow resonance with positive strangeness by LEPS, DIANA, CLAS, SAPHIR and NA49 Collaboration shall be a milestone in the hadron spectroscopy, if these states are further established experimentally. Perhaps, a new landscape is emerging on the horizon, of which we have only had a first glimpse through the above experiments. Now arises a natural question: are there other "genuine" hadrons with valence quark (anti-quark) number $N = 4, 6, 7, 8, \cdots$, in which quarks do not form two or more color-singlet clusters such as hadronic molecules or nuclei? Is there an upper limit for N? In our universe, there may exist quark stars where the quark number is huge. Is there a gap in N from pentaquark states to quark stars? All these are very interesting issues awaiting further experimental exploration.

On the other hand, these experiments have triggered heated discussions of the interpretation of these resonances [10, 11, 12, 13, 14, 15, 16, 18, 19]. Up to now, the parity and angular moment of the Θ^+ particle have not been determined while its isospin has not been cross-checked by other groups.

The partial motivation of the recent experimental search of the pentaquark state came from the work by Diakonov et al. [20]. They proposed the possible existence of the $S = 1$ $J^P = \frac{1}{2}^+$ resonance at 1530 MeV with a width less than 15 MeV using the chiral soliton model and argued that Θ^+ is the lightest member of the anti-decuplet multiplet which is

the third rotational state of the chiral soliton model (CSM). Assuming that the $N(1710)$ is a member of the anti-decuplet, Θ^+ mass is fixed with the symmetry consideration of the model.

However, identifying $N(1710)$ as a member of the anti-decuplet in the CSM is kind of arbitrary [21]. Instead, if the anti-decuplet P_{11} is $N(1440)$, the Θ^+ would be stable as the ground state octet with a very low mass while Θ^+ would be very broad with the anti-decuplet P_{11} being $N(2100)$ [21]. Furthermore, if the decay width of the anti-decuplet $N(1710)$ was shifted upwards to be comparable with PDG values, the predicted width of Θ^+ particle would have exceeded the present experimental upper bound [21]. Moreover, the mass of the pentaquark state with the quark content $(dsds\bar{u})$ is rigorously predicted in the CSM to be 2070 MeV, which is 210 MeV higher than the experimental value measured by NA49 Collaboration [5].

A more serious challenge to the chiral soliton model came from the large N_c consistency consideration by Cohen [15]. He found that predictions for a light collective Θ^+ baryon state (with strangeness $+1$) based on the collective quantization of chiral soliton models are shown to be inconsistent with large N_c QCD since collective quantization is legitimate only for excitations which vanish as $N_c \to \infty$. He concluded that the prediction for Θ^+ properties based on collective quantization of CSM was not valid [15].

The relationship between the bound state and the SU(3) rigid rotator approaches to strangeness in the Skyrme model was investigated in [16]. It was found that the exotic state may be an artifact of the rigid rotator approach to the Skyrme model for large N_c and small m_K.

Jaffe and Wilczek proposed that the observed Θ^+ state could be composed of an anti-strange quark and two highly correlated up and down quark pairs arising from strong color-spin correlation [11]. The resulting J^P of Θ^+ is $\frac{1}{2}^+$. They predicted the isospin 3/2 multiplet of Ξ $(ddss\bar{u})$ with $S = -2$ and $J^{\Pi} = \frac{1}{2}^+$ around 1750 MeV. Such a state with the same quantum number was observed by NA49 but with a much higher mass at 1860 MeV [5].

We have estimated the mass of the pentaquark state with QCD sum rules and found that pentaquark states with isospin $I = 0, 1, 2$ lie close to each other around (1.55 ± 0.15) GeV. Unfortunately we are unable to determine its parity. However, we pointed out that the experimentally observed baryon resonance $\Theta^+(1540)$ with $S = +1$ can be consistently identified as a pentaquark state if its $J^P = \frac{1}{2}^-$. Such a state was expected in QCD. If its parity is positive, this pentaquark state would be really exotic. We emphasized that the outstanding issue is to determine its quantum numbers experimentally.

In the present work, we shall employ the light cone QCD sum rules (LCQSR) to extract the magnetic moment of the Θ^+ particle. The baryon magnetic moment is another fundamental observable as its mass, which encodes information of the underlying quark structure and dynamics. Different models generally predict different values. Such a study will deepen our knowledge of pentaquark states and may help us explore its dynamics and distinguish so many models in the literature.

Our paper is organized as follows: Section I is an introduction. A brief review of this field is presented. In Section II we summarize our previous work on the pentaquark mass sum rule. Then we present the formalism of LCQSR in Section III. Our numerical

analysis and discussions are given in Section IV, where we also compare our result with other groups' prediction.

2 Mass Sum Rule

The method of QCD sum rules incorporates two basic properties of QCD in the low energy domain: confinement and approximate chiral symmetry and its spontaneous breaking. One considers a correlation function of some specific interpolating currents with the proper quantum numbers and calculates the correlator perturbatively starting from high energy region. Then the resonance region is approached where non-perturbative corrections in terms of various condensates gradually become important. Using the operator product expansion, the spectral density of the correlator at the quark gluon level can be obtained in QCD. On the other hand, the spectral density can be expressed in term of physical observables like masses, decay constants, coupling constants etc at the hadron level. With the assumption of quark hadron duality these two spectral densities can be related to each other. In this way one can extract hadron masses etc. For the past two decades QCD sum rule has proven to be a very powerful and successful non-perturbative method [22, 23].

Due to the low mass of Θ^+, we have argued its angular momentum is likely to be one half and considered the correlator for $I = 0$ pentaquark state [10]

$$i \int d^4 x e^{ipx} \langle 0|T\{\eta_0(x), \bar{\eta}_0(0)\}|0 \rangle = \Pi(p)\hat{p} + \Pi'(p) \tag{1}$$

where $\bar{\eta} = \eta^\dagger \gamma_0$ and $\hat{p} = p_\mu \cdot \gamma^\mu$. The interpolating current takes the form [10]

$$\eta_0(x) = \frac{1}{\sqrt{2}} \epsilon^{abc} [u_a^T(x) C\gamma_5 d_b(x)] \{u_e(x)\bar{s}_e(x) i\gamma_5 d_c(x) - (u \leftrightarrow d)\} \tag{2}$$

where a, b, c etc are the color indices. T denotes transpose. C is the charge conjugation matrix. $(C\gamma_5)^T = -C\gamma_5$ ensures the isospin of the up and down quark pair inside the first bracket to be zero. The anti-symmetrization in the second bracket ensures that the isospin of the other up and down quark pair is also zero.

The overlapping amplitude f_0 of the interpolating current with the pentaquark state was defined as

$$\langle 0|\eta_0(0)|p, I = 0 \rangle = f_0 u(p) \tag{3}$$

where $u(p)$ is the Dirac spinor of pentaquark field with $I = 0$.

At the hadron level, the chiral even structure $\Pi(p)$ can be expressed as

$$\Pi(p) = \frac{f_0^2}{p^2 - M_0^2} + \text{higher states} \tag{4}$$

where M_0 is the pentaquark mass. On the other hand, it will be calculated in terms of quarks and gluons.

$$\langle 0|T\{\eta_0(x)\bar{\eta}_0(0)\}|0 \rangle = -\epsilon^{abc}\epsilon^{a'b'c'} \times$$

4

$$\{-\mathbf{Tr}\left[iS_d^{bb'}(x)\gamma_5 C iS_u^{Taa'}(x)C\gamma_5\right]\mathbf{Tr}\left[i\gamma_5 iS_d^{cc'}(x)i\gamma_5 iS_s^{ee'}(-x)\right]iS_u^{ee'}(x)$$

$$+\mathbf{Tr}\left[i\gamma_5 iS_d^{cc'}(x)i\gamma_5 iS_s^{ee'}(-x)\right]iS_u^{ea'}(x)\gamma_5 C iS_d^{Tbb'}(x)C\gamma_5 iS_u^{ae'}(x)$$

$$-\mathbf{Tr}\left[iS_d^{bb'}(x)i\gamma_5 iS_s^{ee'}(-x)i\gamma_5 iS_d^{cc'}(x)\gamma_5 C iS_u^{Taa'}(x)C\gamma_5\right]iS_u^{ee'}(x)$$

$$+iS_u^{ea'}(x)\gamma_5 C\left[iS_d^{bb'}(x)i\gamma_5 iS_s^{ee'}(-x)i\gamma_5 iS_d^{cc'}(x)\right]^T C\gamma_5 iS_u^{ae'}(x)$$

$$-\mathbf{Tr}\left[iS_d^{bb'}(x)\gamma_5 C iS_u^{Taa'}(x)C\gamma_5\right]iS_u^{ec'}(x)i\gamma_5 iS_s^{ee'}(-x)i\gamma_5 iS_d^{ce'}(x)$$

$$+iS_u^{ec'}(x)i\gamma_5 iS_s^{ee'}(-x)i\gamma_5 iS_d^{cb'}(x)\gamma_5 C iS_u^{Taa'}(x)C\gamma_5 iS_d^{be'}(x)$$

$$+iS_u^{ea'}(x)\gamma_5 C iS_d^{Tbb'}(x)C\gamma_5 iS_u^{ac'}(x)i\gamma_5 iS_s^{ee'}(-x)i\gamma_5 iS_d^{ce'}(x)$$

$$+iS_u^{ea'}(x)\gamma_5 C\left[iS_u^{ac'}(x)i\gamma_5 iS_s^{ee'}(-x)i\gamma_5 iS_d^{cb'}(x)\right]^T C\gamma_5 iS_d^{be'}(x)\} \tag{5}$$

where $iS_s^{ee'}(-x)$ is the strange quark propagator in the coordinate space.

After making Fourier transformation to the above equation and invoking Borel transformation to Eq. (1) we have obtained the mass sum rule [10]

$$f_0^2 e^{-\frac{M_0^2}{M^2}} = \int_{m_s^2}^{s_0} e^{-\frac{s}{M^2}}\rho_0(s)ds \tag{6}$$

where m_s is the strange quark mass, $\rho_0(s)$ is the spectral density and s_0 is the threshold parameter used to subtract the higher state contribution with the help of quark-hadron duality assumption. Roughly speaking, $\sqrt{s_0}$ is around the first radial excitation mass. The spectral density reads

$$\rho_0 = \frac{1}{(2\pi)^8}[\frac{3s^5}{2^8 7!} + \frac{s^2}{96}(\frac{5}{12}a_q^2 + \frac{11}{24}a_s a_q) + (\frac{7}{432}a_s a_q^3 + \frac{1}{864}a_q^4)\delta(s)] \tag{7}$$

where we have used the factorization approximation for the multi-quark condensates.

The pentaquark mass was found to be [10]

$$M_0^2 = \frac{\int_{m_s^2}^{s_0} e^{-s/M^2}\rho'(s)ds}{\int_{m_s^2}^{s_0} e^{-s/M^2}\rho(s)}ds \tag{8}$$

with $\rho'(s) = s\rho(s)$ except that $\rho'(s)$ does not contain the last term in $\rho(s)$.

In the numerical analysis, we have used the values of various QCD condensates $a_q = -(2\pi)^2\langle\bar{q}q\rangle = 0.55\text{GeV}^3$, $a_s = 0.8a_q = 0.44\text{GeV}^3$. We used $m_s(1\text{GeV}) = 0.15$ GeV for the strange quark mass in the $\bar{MS}$ scheme. Numerically we arrived at $M_0 = (1.56\pm0.15)\text{GeV}$, where the central value corresponds to $M^2 = 2$ GeV^2 and $s_0 = 4$ GeV^2.

3 Formalism of Light Cone QCD Sum Rules

The LCQSR is quite different from the conventional mass QSR, which is based on the short-distance operator product expansion (OPE). The LCQSR is based on the OPE on the light cone, which is the expansion over the twists of the operators [24]. The main contribution comes from the lowest twist operator. Matrix elements of nonlocal operators

sandwiched between a photon (or hadronic state) and the vacuum define the photon (hadron) distribution amplitudes. When the LCQSR is used to calculate the coupling constant, the double Borel transformation is always invoked so that the excited states and the continuum contribution can be treated quite nicely. Moreover, the final sum rule depends only on the value of the photon (or hadron) distribution amplitude at a specific point, which is much better known than the whole distribution function. In the present case our sum rule involves the photon light cone distribution amplitudes $\varphi_\gamma(u_0 = \frac{1}{2})$. Note this parameter is universal in all processes at a given scale. In this respect, $\varphi_\gamma(u_0 = \frac{1}{2})$ is a fundamental quantity like the quark condensate, which is to be determined with various non-perturbative methods. Like the quark condensate, it can be extracted consistently through the analysis of the light cone sum rules.

In the framework of QCD sum rules, the nucleon magnetic moment was first studied using the external field method in Refs. [25, 26, 27]. The presence of the electromagnetic field will polarize the vacuum and lead to a few new induced condensates with various universal vacuum susceptibilities. Later this formalism was extended to extract the magnetic moments of the octet and decuplet baryon and heavy baryons [28, 29]. LCQSRs were first used to calculate the magnetic moments of nucleons in [30]. Recently the magnetic moments of the octet and decuplet baryons were reformulated and discussed with the help of the light cone QCD sum rule technique [31].

In the present case, we are interested in the pentaquark magnetic moments. We shall consider the following correlator

$$\Pi(p_1, p_2, q) = i \int d^4x \, e^{ipx} \langle \gamma(q) | T\{\eta_0(x)\bar{\eta}_0(0)\} | 0 \rangle \tag{9}$$

where γ represents the external electromagnetic field with the vector potential $B_\mu(x) = \varepsilon_\mu e^{iq\cdot x}$. ε_μ is the photon polarization vector. Throughout this work we shall use the convention of outgoing photons. Its field strength is $F_{\mu\nu}(x) = (-i)(\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu)e^{iq\cdot x}$. $p_1 = p, p_2 = p_1 + q$ is the final and initial pentaquark momentum.

At the hadron level, the correlator can be expressed as

$$\begin{aligned}
\Pi(p_1, p_2, q) = \quad & f_0^2 \varepsilon^\mu \frac{\not{p}_1 + m_0}{p_1^2 - m_0^2} [F_1(q^2)\gamma_\mu + \frac{i\sigma_{\mu\nu}q^\nu}{2m_0} F_2(q^2)] \frac{\not{p}_2 + m_0}{p_2^2 - m_0^2} \\
& + f_0 f_* \varepsilon^\mu \frac{\not{p}_1 + m_0}{p_1^2 - m_0^2} [F_1^*(q^2)\gamma_\mu + \frac{i\sigma_{\mu\nu}q^\nu}{m_0 + m_*} F_2^*(q^2)] \frac{\not{p}_2 + m_*}{p_2^2 - m_*^2} \\
& + f_* f_0 \varepsilon^\mu \frac{\not{p}_1 + m_*}{p_1^2 - m_*^2} [F_1^*(q^2)\gamma_\mu + \frac{i\sigma_{\mu\nu}q^\nu}{m_0 + m_*} F_2^*(q^2)] \frac{\not{p}_2 + m_0}{p_2^2 - m_0^2} + \cdots
\end{aligned} \tag{10}$$

where m_* is the mass of the excited pentaquark state, f_* is the overlapping amplitude of our interpolating current with these states. $F_{1,2}^*(q^2)$ are the electromagnetic transition form factors between the ground state and excited pentaquarks.

We have used the electromagnetic vertex of Θ^+ in writing down the above formula.

$$\langle \Theta^+(p_1) | \Theta^+(p_2) \rangle_\gamma = \varepsilon^\mu \bar{u}_0(p_1)[F_1(q^2)\gamma_\mu + \frac{i\sigma_{\mu\nu}q^\nu}{2m_0} F_2(q^2)] u_0(p_2) \tag{11}$$

In Eq. (10) the first term contains two poles at both $p_1^2 = m_0^2$ and $p_2^2 = m_0^2$ with both the initial and final baryon being the ground state pentaquark. We have also explicitly written down terms with a single pole either at $p_1^2 = m_0^2$ or $p_2^2 = m_0^2$. In this case one of the

initial or final state is the excited state. The ellipse denotes the continuum contribution. As we will show below, the contribution from all the non-diagonal terms in Eq. (10) will be either eliminated or strongly suppressed after we invoke double Borel transformation with the variables p_1^2, p_2^2 simultaneously.

Rewriting Eq. (10) we get

$$
\begin{aligned}
\Pi(p_1, p_2, q) &= f_0^2 \varepsilon^\mu \frac{\not{p}_1 + m_0}{p_1^2 - m_0^2} [(F_1(q^2) + F_2(q^2))\gamma_\mu + \frac{(p_1+p_2)_\mu}{2m_0} F_2(q^2)] \frac{\not{p}_2 + m_0}{p_2^2 - m_0^2} + \cdots \\
&= \frac{f_0^2}{(p_1^2 - m_0^2)(p_2^2 - m_0^2)} [F_1(q^2) + F_2(q^2)] \not{p}_1 \not{\varepsilon} \not{p}_2 + \cdots
\end{aligned}
\tag{12}
$$

The pentaquark magnetic moment is defined as

$$
\mu_{\Theta^+} = [F_1(0) + F_2(0)] \frac{e_{\Theta^+}}{2m_0}
\tag{13}
$$

We are only interested in the term involving the magnetic form factor $F_1(q^2) + F_2(q^2)$. So we focus on the tensor structure $\not{p}_1 \not{\varepsilon} \not{p}_2$, which is equivalent to $-i\epsilon_{\mu\nu\alpha\beta}\gamma^\mu\gamma_5\varepsilon^\nu q^\alpha p^\beta$ up to terms containing a single gamma matrix.

At the quark gluon level, the expression of the above correlator can be obtained through simple replacement in Eq. (5). There are two classes of diagrams according to the way how the photon couples to the quark lines. First, the photon couples to the quark line perturbatively through the standard QED interaction. For this set of diagrams, we may replace one of the free quark propagator in Eq. (5) by the one with the electromagnetic interaction

$$
\langle 0|T\{q^a(x)\bar{q}^b(0)\}|0\rangle_{F_{\mu\nu}} = \frac{\delta^{ab}e_q}{16\pi^2 x^2} \int_0^1 du\{2(1-2u)x_\mu\gamma_\nu + i\epsilon_{\mu\nu\rho\sigma}\gamma_5\gamma^\rho x^\sigma\}F^{\mu\nu}(ux)
\tag{14}
$$

$$
\langle 0|T\{q^a(0)\bar{q}^b(x)\}|0\rangle_{F_{\mu\nu}} = -\frac{\delta^{ab}e_q}{16\pi^2 x^2} \int_0^1 du\{2(1-2u)x_\mu\gamma_\nu + i\epsilon_{\mu\nu\rho\sigma}\gamma_5\gamma^\rho x^\sigma\}F^{\mu\nu}[(1-u)x]
\tag{15}
$$

where we have adopted the Fock-Schwinger gauge $x^\mu A_\mu(x) = 0$ to express the electromagnetic vector potential in terms of the gauge invariant $F_{\mu\nu}$.

The second class of diagrams involve the non-perturbative interaction of photons with the quarks in terms of the photon light cone distribution amplitude. One of the five propagators in Eq. (5) is substituted by

$$
\begin{aligned}
\langle \gamma(q)|q^a(x)\bar{q}^b(0)|0\rangle = &-\frac{\sigma^{\mu\nu}}{8}\langle\gamma(q)|\bar{q}^b(0)\sigma_{\mu\nu}q^a(x)|0\rangle + \frac{\gamma^\mu\gamma_5}{4}\langle\gamma(q)|\bar{q}^b(0)\gamma_\mu\gamma_5 q^a(x)|0\rangle \\
&-\frac{\gamma^\mu}{4}\langle\gamma(q)|\bar{q}^b(0)\gamma_\mu q^a(x)|0\rangle
\end{aligned}
\tag{16}
$$

The two-particle photon light cone distribution amplitudes (LCPDA) are defined as [24, 32]:

$$
\begin{aligned}
\langle \gamma(q)|\bar{q}(x)\sigma_{\mu\nu}q(0)|0\rangle = &\ ie_q\langle\bar{q}q\rangle \int_0^1 du\, e^{iuqx}((\varepsilon_\alpha q_\beta - \varepsilon_\beta q_\alpha)\{\chi\varphi(u) + x^2[(g_1(u) \\
&-g_2(u)]\} + \{qx(\varepsilon_\alpha x_\beta - \varepsilon_\beta x_\alpha) + \varepsilon x(x_\alpha q_\beta - x_\beta q_\alpha)\}g_2(u))
\end{aligned}
\tag{17}
$$

$$
\langle \gamma(q)|\bar{q}(x)\gamma_\mu\gamma_5 q(0)|0\rangle = \frac{f}{4}e_q\epsilon_{\mu\nu\rho\sigma}\varepsilon^\nu q^\rho x^\sigma \int_0^1 du\, e^{iuqx}\psi(u)
\tag{18}
$$

7

$$\langle\gamma(q)|\bar{q}(x)\gamma_\mu q(0)|0\rangle = f^{(V)}e_q\varepsilon_\mu\int_0^1 du e^{iuqx}\psi^{(V)}(u) \tag{19}$$

The $\varphi(u)$ is associated with the leading twist-two photon wave function, while $g_1(u)$ and $g_2(u)$ are twist-4 LCPDAs. All these LCPDAs are normalized to unity, $\int_0^1 du\, f(u) = 1$. In the above formula, the summation over quark color indices is implicitly assumed.

With these definitions and spatial translation transformation, it's easy to derive

$$\langle\gamma(q)|\bar{q}(0)\sigma_{\mu\nu}q(x)|0\rangle = \quad ie_q\langle\bar{q}q\rangle\int_0^1 du\, e^{i(1-u)qx}((\varepsilon_\alpha q_\beta - \varepsilon_\beta q_\alpha)\{\chi\varphi(u) + x^2[(g_1(u)$$
$$-g_2(u)]\} + \{qx(\varepsilon_\alpha x_\beta - \varepsilon_\beta x_\alpha) + \varepsilon x(x_\alpha q_\beta - x_\beta q_\alpha)\}g_2(u)) \tag{20}$$

$$\langle\gamma(q)|\bar{q}(0)\gamma_\mu\gamma_5 q(x)|0\rangle = -\frac{f}{4}e_q\epsilon_{\mu\nu\rho\sigma}\varepsilon^\nu q^\rho x^\sigma\int_0^1 due^{i(1-u)qx}\psi(u) \tag{21}$$

$$\langle\gamma(q)|\bar{q}(0)\gamma_\mu q(x)|0\rangle = f^{(V)}e_q\varepsilon_\mu\int_0^1 due^{i(1-u)qx}\psi^{(V)}(u) \tag{22}$$

After tedious but straightforward calculation we arrive at the correlator in the coordinate space, to which we then make Fourier transformation. The formulas are:

$$\int\frac{e^{ipx}}{(x^2)^n}d^Dx = i(-1)^{n+1}\frac{2^{D-2n}\pi^{D/2}}{(-p^2)^{D/2-n}}\frac{\Gamma(D/2-n)}{\Gamma(n)}\ , \tag{23}$$

$$\int\frac{\hat{x}e^{ipx}}{(x^2)^n}d^Dx = (-1)^{n+1}\frac{2^{D-2n+1}\pi^{D/2}}{(-p^2)^{D/2+1-n}}\frac{\Gamma(D/2+1-n)}{\Gamma(n)}\hat{p}\ . \tag{24}$$

After isolating the correct tensor structure, we further make double Borel transformation with the variables p_1^2 and p_2^2. In this way the single-pole terms in (12) are eliminated. The formula reads:

$$\mathcal{B}_1{}^{M_1^2}_{p_1^2}\mathcal{B}_2{}^{M_2^2}_{p_2^2}\frac{\Gamma(n)}{[m^2-(1-u)p_1^2-up_2^2]^n} = (M^2)^{2-n}e^{-\frac{m^2}{M^2}}\delta(u-u_0)\ . \tag{25}$$

Here $M = \frac{M_1^2 M_2^2}{M_1^2+M_2^2}$ is the Borel parameter and $u_0 \equiv \frac{M_1^2}{M_1^2+M_2^2}, 1-u_0 \equiv \frac{M_2^2}{M_1^2+M_2^2}$.

Subtracting the continuum contribution which is modelled by the dispersion integral in the region $s_1, s_2 \geq s_0$, we arrive at:

$$(2\pi)^8 f_0^2|[F_1(0)+F_2(0)]|e^{-\frac{M_0^2}{M^2}} = -\{\frac{\pi^2}{5!2^4}(14e_u+3e_d)f\psi(1-u_0)M^{10}f_4(\frac{s_0}{M^2})$$
$$+\frac{\pi^2}{5!2^3}e_s f\psi(u_0)M^{10}f_4(\frac{s_0}{M^2}) - \frac{\pi^2}{3^2 2^6}(14e_u+3e_d)fm_s a_s\psi(1-u_0)M^6 f_2(\frac{s_0}{M^2})$$
$$+\frac{\pi^2}{3^2 2^5}(20e_u+5e_d)fm_s a_q\psi(1-u_0)M^6 f_2(\frac{s_0}{M^2})$$
$$+\frac{\pi^2}{3^2 2^5}(20e_u+5e_d)fa_s a_q\psi(1-u_0)M^4 f_1(\frac{s_0}{M^2})$$
$$+\frac{\pi^2}{3^2 2^5}(20e_u+5e_d)fa_q^2\psi(1-u_0)M^4 f_1(\frac{s_0}{M^2})$$
$$+\frac{\pi^2}{3^2 2^2}e_s fa_q^2\psi(u_0)M^4 f_1(\frac{s_0}{M^2}) - \frac{1}{5!2^6}(14e_u+3e_d+2e_s)M^{12}f_5(\frac{s_0}{M^2})$$
$$+\frac{1}{3\times 2^9}(14e_u+3e_d)m_s a_s M^8 f_3(\frac{s_0}{M^2}) - \frac{1}{3\times 2^8}(20e_u+5e_d)m_s a_q M^8 f_3(\frac{s_0}{M^2})$$

8

$$-\frac{1}{3^2 2^6}(20e_u + 5e_d)a_s a_q M^6 f_2(\tfrac{s_0}{M^2}) - \frac{1}{3^2 2^6}(20e_u + 5e_d + 8e_s)a_q^2 M^6 f_2(\tfrac{s_0}{M^2})$$

$$-\frac{1}{3^3 2^5}(14e_u + 3e_d)a_s a_q^3 - \frac{1}{3^3 2^4}e_s a_q^4 + \frac{1}{5! 2^5}(e_u + 2e_d)m_s a_q \chi \varphi(1-u_0)M^{10}f_4(\tfrac{s_0}{M^2})$$

$$-\frac{1}{3\times 2^6}(e_u + 2e_d)m_s a_q[g_1(1-u_0) - g_2(1-u_0)]M^8 f_3(\tfrac{s_0}{M^2})$$

$$+\frac{1}{3^2 2^8}(e_u + 2e_d)a_s a_q \chi \varphi(1-u_0)M^8 f_3(\tfrac{s_0}{M^2})$$

$$-\frac{1}{3^2 2^4}(e_u + 2e_d)a_s a_q[g_1(1-u_0) - g_2(1-u_0)]M^6 f_2(\tfrac{s_0}{M^2})$$

$$-\frac{1}{3\times 2^8}(e_u - 2e_d)a_q^2 \chi \varphi(1-u_0)M^8 f_3(\tfrac{s_0}{M^2})$$

$$+\frac{1}{3\times 2^4}(e_u - 2e_d)a_q^2[g_1(1-u_0) - g_2(1-u_0)]M^6 f_2(\tfrac{s_0}{M^2})$$

$$+\frac{1}{3\times 2^7}e_s a_s a_q \chi \varphi(u_0)M^8 f_3(\tfrac{s_0}{M^2})$$

$$-\frac{1}{3\times 2^3}e_s a_s a_q[g_1(u_0) - g_2(u_0)]M^6 f_2(\tfrac{s_0}{M^2}) - \frac{1}{3^2 2^5}(e_u - 2e_d)a_s a_q^3 \chi \varphi(1-u_0)M^2 f_0(\tfrac{s_0}{M^2})$$

$$+\frac{1}{3^2 2^4}e_s a_s a_q^3 \chi \varphi(u_0)M^2 f_0(\tfrac{s_0}{M^2}) + \frac{1}{3^2 2^3}(e_u - 2e_d)a_s a_q^3[g_1(1-u_0) - g_2(1-u_0)]$$

$$-\frac{1}{3^2 2^2}e_s a_s a_q^3[g_1(u_0) - g_2(u_0)] + \frac{1}{3^3 2^5}(e_u + 2e_d)a_q^4 \chi \varphi(1-u_0)M^2 f_0(\tfrac{s_0}{M^2})$$

$$-\frac{1}{3^3 2^3}(e_u + 2e_d)a_q^4[g_1(1-u_0) - g_2(1-u_0)]$$

$$+\frac{1}{3\times 2^7}(e_u - 2e_d)m_s a_s a_q^2 \chi \varphi(1-u_0)M^4 f_1(\tfrac{s_0}{M^2})$$

$$-\frac{1}{3\times 2^4}(e_u - 2e_d)m_s a_s a_q^2[g_1(1-u_0) - g_2(1-u_0)]M^2 f_0(\tfrac{s_0}{M^2})$$

$$-\frac{1}{3\times 2^6}(e_u - 2e_d)m_s a_q^3 \chi \varphi(1-u_0)M^4 f_1(\tfrac{s_0}{M^2})$$

$$+\frac{1}{3\times 2^3}(e_u - 2e_d)m_s a_q^3[g_1(1-u_0) - g_2(1-u_0)]M^2 f_0(\tfrac{s_0}{M^2})\} \tag{26}$$

where we have used the functions $f_n(x) = 1 - e^{-x}\sum_{k=0}^{n}\frac{x^k}{k!}$ to subtract the excited states and continuum contribution.

The left side of the above equation is obtained from double Borel transformation to the hadron level correlator. We have assumed the quark-hadron duality. Since in the present case, both the initial and final states are the same pentaquark. It is natural to employ $M_1^2 = M_2^2 = 2M^2$ so we have $u_0 = \frac{1}{2}$.

4   Results and Discussions

Dividing our light cone sum rule for the pentaquark magnetic moment Eq. (26) by its mass sum rule Eq. (6), we can extract the combination $|F_1(0) + F_2(0)|$. In the numerical analysis, we follow Refs. [24, 32] and use the following form for the photon light cone distribution amplitudes,

$$\psi(u) = 1,$$
$$\varphi(u) = 6u(1-u),$$
$$g_1(u) = -\frac{1}{8}(1-u)(3-u),$$
$$g_2(u) = -\frac{1}{4}(1-u)^2$$

with $f = 0.028\text{GeV}^2$, $\chi(1\text{GeV}) = -4.4\text{GeV}^{-2}$ [25, 26, 27, 24, 32].

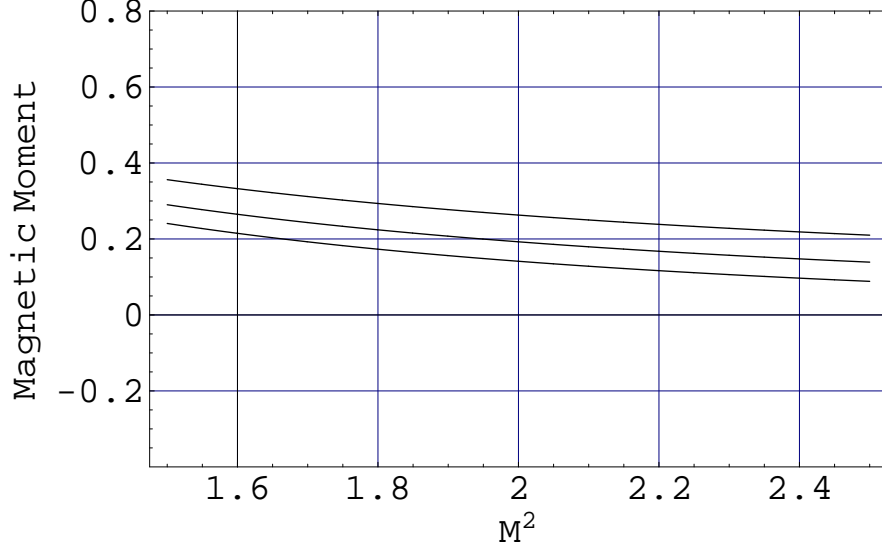The variation of the pentaquark magnetic moment with M^2, s_0 is shown in Figure 1.

Figure 1: The variation of the absolute value of the magnetic moment of Θ^+ (in unit of $\frac{e_{\Theta^+}}{2m_0}$), with the Borel parameter M^2 and the continuum threshold s_0. The three curves in this figure (from top to bottom) correspond to $s_0 = 3.61, 4.0, 4.41\text{GeV}^2$ respectively.

Numerically we get

$$|\mu_{\Theta^+}| = (0.20 \pm 0.10)\frac{e_{\Theta^+}}{2m_0}, \tag{27}$$

In unit of nucleon magneton, we have

$$|\mu_{\Theta^+}| = (0.12 \pm 0.06)\mu_N, \tag{28}$$

The central value is obtained at $M^2 = 2.0\text{GeV}^2$ and $s_0 = 4.0$ GeV^2. The errors come from (1) the uncertainty of the values of the photon light cone distribution amplitudes at $u_0 = \frac{1}{2}$; (2) the truncation of the expansion over the twist and keeping only the lowest-twist few terms containing two particles; (3) the truncation of the operator product expansion in the calculation of the terms not involving the photon LCPDA; (4) the uncertainty of the condensate values; (5) the variation of the sum rule with the continuum threshold and the Borel parameter within the working interval; (6) the neglect of the higher dimension condensates; (7) the neglect of perturbative QCD corrections etc.

In our calculation we have assumed the pentaquark state Θ^+ is an isoscalar with $J = \frac{1}{2}$. No assumption is made of its parity. In fact, our interpolating current couples to pentaquark with both negative and positive parity as pointed out in Ref. [10]. Our formalism picks out only the state with lowest mass without knowledge of its parity. Fortunately the electromagnetic vertex of pentaquarks with either negative or positive parity is the same, which ensures that we can extract the absolute value of the magnetic moment of the lowest pentaquark state even if we do not know its parity. Our result shows that the magnetic moment of Θ^+ is quite small.

In Ref. [17], Nam, Hosaka and Kim first estimated the anomalous magnetic moment κ_{Θ^+} of the Θ^+ pentaquark when they discussed the photo-production of Θ^+ from the

10

neutron. For example, they have estimated $\kappa_{\Theta^+} = -0.7$ and $\kappa_{\Theta^+} = -0.2$ for $J^P = \frac{1}{2}^+$ and $J^P = \frac{1}{2}^-$ respectively using Jaffe and Wilczek's diquark picture. Also, they have considered the possibility of a K N bound system, with $\kappa(KN, J^P = \frac{1}{2}^+) = -0.4$ and $\kappa(KN, J^P = \frac{1}{2}^-) = -0.5$. These numbers are all in unit of Θ^+ magneton $\frac{e_0}{2m_{\Theta^+}}$.

In Ref. [18], a quark model calculation of the Θ^+ magnetic moment was performed using Jaffe and Wilczek's picture for the pentaquark [11]. For comparison, we borrow the relevant two formulae from Ref. [18] and list them below. First Zhao wrote down [18]:

$$\mu_\theta s_z(s_z = \frac{1}{2}) = \langle \Theta^+ | \left[\mu_{ud}\mathbf{0} + \mu_{ud}\mathbf{0} + \mu_{\bar{s}}\frac{\mathbf{1}}{\mathbf{2}} + \frac{e_{ud}}{2m_{ud}}\mathbf{1} \right] | \Theta^+ \rangle (s_z = \frac{1}{2}), \qquad (29)$$

with the bold numbers denoting the spin and orbital angular momentum vector. We note in passing that the author has made an implicit assumption that ud quark pairs are point-like and there is no quark exchange between these two pairs. Otherwise the above simple formula may not hold.

Then Zhao obtained [18]

$$\frac{1}{2}\mu_\theta = \frac{1}{2}a^2 \left[\langle 1\ 0, \frac{1}{2}\frac{1}{2} | \frac{1}{2}\frac{1}{2} \rangle^2 - \langle 1\ 1, \frac{1}{2} - \frac{1}{2} | \frac{1}{2}\frac{1}{2} \rangle^2 \right] \mu_{\bar{s}} + b^2 \langle 1\ 1, \frac{1}{2} - \frac{1}{2} | \frac{1}{2}\frac{1}{2} \rangle^2 \frac{e_{ud}}{2m_{ud}}, \qquad (30)$$

With $\mu_{\bar{s}} \equiv e_{\bar{s}}/2m_s$, $m_s = 500$ MeV, $e_{ud} \equiv e_u + e_d = e_0/3$, $m_{ud} = 720$ MeV and assuming equal probability for two spatial configuration, i.e., $a = b = \frac{1}{\sqrt{2}}$, he finally obtained [18]

$$\mu_{\Theta^+} = 0.13 \frac{e}{2m_0} \qquad (31)$$

In Ref. [19] Kim and Praszalowicz derived relations for the anti-decuplet within the framework of chiral soliton model in the chiral limit. The Θ^+ pentaquark magnetic moment is estimated to be $(0.2 \sim 0.3)\mu_N$ [19]. It's interesting to note that magnetic moment of θ pentaquark derived from different models is also quite small.

In short summary, we have estimated the magnetic moment of the Θ^+ pentaquark state in the framework of light cone QCD sum rules using the photon distribution amplitude. All the necessary parameters in the present calculation have been determined from previous studies. To this extent, our calculation may be viewed as a robust prediction.

The width of the Θ^+ pentaquark is so narrow. With the accumulation of events, its magnetic moment may be extracted from experiments eventually in the near future, which may help distinguish different theoretical models and deepen our understanding of the underlying dynamics governing its formation.

References

[1] T. Nakano et al., Phys. Rev. Lett. 91, 012002 (2003).

[2] V. V. Barmin et al., hep-ex/0304040.

[3] S. Stepanyan et al., hep-ex/0307018.

[4] J. Barth et al., hep-ph/0307083.

[5] NA49 Collaboration, hep-ex/0310014.

[6] S. J. Dong et al., hep-ph/0306199.

[7] S. Godfrey and N. Isgur, Phys. Rev. D 32, 189 (1985);S. Capstick and N. Isgur, Phys. Rev. D 34, 2809 (1986).

[8] T. D. Cohen, Phys. Lett. B 427, 348 (1998).

[9] Particle Dada Group, Phys. Rev. D 66, 010001 (2002).

[10] Shi-Lin Zhu, hep-ph/0307345, Phys. Rev. Lett. 91, 232002 (2003).

[11] R. Jaffe and F. Wilczek, hep-ph/0307341, Phys. Rev. Lett. 91, 232003 (2003).

[12] H. Gao and B.-Q. Ma, hep-ph/0305294, Mod. Phys. Lett. A 14, 2313 (1999);M. V. Ployakov and A. Rathke, hep-ph/0303138;F. Stancu and D. O. Riska, hep-ph/0307010; B. G. Wybourne, hep-ph/0307170; A. Hosaka, hep-ph/0307232; T. Hyodo, A. Hosaka, E. Oset, nucl-th/0307105; M. Karliner and H. J. Lipkin, hep-ph/0307243, hep-ph/0307343.

[13] P.V. Pobylitsa, hep-ph/0310221; D. Diakonov, V. Petrov, hep-ph/0310212; J. Letessier, G. Torrieri, S. Steinke, J. Rafelski, hep-ph/0310188; I.M.Narodetskii et al., hep-ph/0310118; N. Auerbach, V. Zelevinsky, nucl-th/0310029; D.E. Kahana, S.H. Kahana, hep-ph/0310026; J. Haidenbauer, G. Krein, hep-ph/0309243; D. Diakonov, V. Petrov, hep-ph/0309203;L. Ya. Glozman, hep-ph/0309092, hep-ph/0308232; M. Praszalowicz, hep-ph/0308114; R.A. Arndt, I.I. Strakovsky, R.L. Workman, nucl-th/0308012; X. Chen, Y. Mao, B.-Q. Ma, hep-ph/0307381; S. Nussinov, hep-ph/0307357; J. Randrup, nucl-th/0307042; M.V. Polyakov, A. Rathke, hep-ph/0303138; H. Walliser, V.B. Kopeliovich, hep-ph/0304058.

[14] R. Bijker, M.M. Giannini, E. Santopinto, hep-ph/0310281; Y. Oh, H. Kim, S. H. Lee, hep-ph/0310117, hep-ph/0310019, hep-ph/0311054; R.D. Matheus et al., hep-ph/0309001; F.Huang, Z.Y.Zhang, Y.W.Yu, B.S.Zou, hep-ph/0310040; C. E. Carlson et al., hep-ph/0310038, hep-ph/0307396; J. Sugiyama, T. Doi, M. Oka, hep-ph/0309271; F.Csikor, Z. Fodor, S.D. Katz, T.G. Kovacs, hep-lat/0309090; S. Sasaki, hep-lat/0310014; B. Jennings, K. Maltman, hep-ph/0308286; K. Cheung, hep-ph/0308176; P.Bicudo, G. M. Marques, hep-ph/0308073; L. W. Chen, V. Greco, C. M. Ko, S. H. Lee, W. Liu, nucl-th/0308006;W. Liu, C. M. Ko, V. Kubarovsky, nucl-th/0310087; W. Liu, C. M. Ko, nucl-th/0309023, nucl-th/0308034; D. Borisyuk, M. Faber, A. Kobushkin, hep-ph/0307370; Felipe J. Llanes-Estrada, E. Oset, V. Mateu, nucl-th/0311020.

[15] T. D. Cohen, R. F. Lebed, hep-ph/0309150; T. D. Cohen, hep-ph/0309111.

[16] N. Itzhaki et al., hep-ph/0309305.

[17] S.I.Nam, A.Hosaka, H.-Ch.Kim, hep-ph/0308313.

[18] Q. Zhao, hep-ph/0310350.

[19] H.-C. Kim and M. Praszalowicz, hep-ph/0308242.

[20] D. Diakonov, V. Petrov, and M. Ployakov, Z. Phys. A 359, 305 (1997).

[21] S. Capstick, P. R. Page, and W. Roberts, hep-ph/0307019.

[22] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. **B 147**, 385 (1979).

[23] L. J. Reinders, H. Rubinstein, and S. Yazaki, Phys. Rept. 127, 1 (1985).

[24] I. I. Balitsky, V. M. Braun, and A. V. Kolesnichenko, Nucl. Phys. B 312, 509 (1989).

[25] B. L. Ioffe and A. V. Smilga, Nucl. Phys. B **232**, 109 (1984).

[26] I. I. Balitsky and A. V. Yung, Phys. Lett. B **129**, 328 (1983); I. I. Balitsky and A. V. Kolesnichenko, Yad. Fiz. **41**, 282 (1985).

[27] V. M. Belyaev and Ya. I. Kogan, Yad. Fiz **40**, 1035 (1984). (Sov. J. Nucl. Phys. **40**, 659 (1984).)

[28] C. B. Chiu, J. Pasupathy, and S. J. Wilson, Phys. Rev. D **33**, 1961 (1986) ; C. B. Chiu, S. L. Wilson, J. Pasupathy, and J. P. Singh, Phys. Rev. D **36**, 1451, 1553 (1987).

[29] Shi-Lin Zhu, W-Y. P. Hwang and Ze-Sen Yang, Phys. Rev. D **57**, 1527 (1998); *ibid.* D**56**, 7273 (1997); Mod. Phys. Lett.A 12, 3027 (1997); Phys. Lett. B 420, 8 (1998); Shi-Lin Zhu, Yuan-Ben Dai, Phys. Rev. D 59, 114015 (1999); Shi-Lin Zhu, Phys. Rev. D 61, 114019 (2000).

[30] V. M. Braun and I. E. Filyanov, Z. Phys. C **44**, 157 (1989).

[31] T.M. Aliev, I. Kanik, M. Savci, Phys. Rev. D 68, 056002 (2003); T.M. Aliev, A. Ozpineci, M. Savci. Phys. Rev. D 66, 016002 (2002),Erratum-ibid. D 67, (039901) 2003; Phys. Rev. D 65, 096004 (2002); Phys. Rev. D 65, 056008 (2002); Phys. Lett. B 516, 299 (2001); Nucl. Phys. A 678, 443 (2000); T.M. Aliev and A. Ozpineci. Phys. Rev. D 62, 053012 (2000).

[32] V. M. Braun and I. E. Filyanov, Z. Phys. C 48, 239 (1990); A. Ali and V. M. Braun, Phys. Lett. B 359, 223 (1995);G. Eilam, I. Halperin and R. R. Mendel, Phys. Lett. B **361**, 137 (1995); P. Ball, V. M. Braun and N. Kivel, Nucl. Phys. B **649**, 263 (2003).